Micro III - August 2017, Solution Guide

1. Consider the following game G, where Player 1 chooses the row and Player 2 simultaneously chooses the column.

		Player 2		
		D	E	F
	A	$2,2$	$0,1$	$0,3$
Player 1	B	$3,5$	$-1,3$	$2,4$
	C	$2,1$	$2,0$	$2,1$

(a) Show which strategies in G are eliminated by the procedure of 'Iterated Elimination of Strictly Dominated Strategies'.

SOLUTION: E is strictly dominated by D and F for Player 2, and can therefore be eliminated. After eliminating E, A is strictly dominated by B for Player 1, and can therefore be eliminated. No other strategy is strictly dominated for either player.

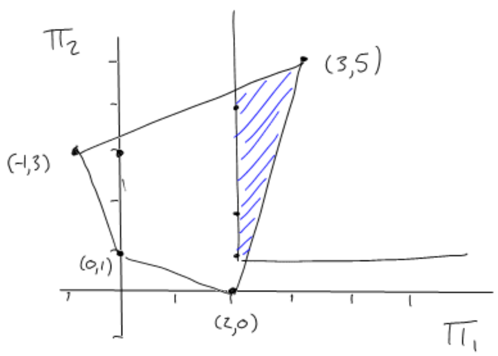
(b) Find all Nash equilibria (NE), pure and mixed, in G. Which NE gives the highest payoff to both players? Denote this equilibrium strategy profile by $e(1)$.

SOLUTION: There are two NE, both in pure strategies: (B, D) *and* (C, F)*, where the former gives a higher payoff to both players than the latter. Notice that after carrying out IESDS, C is weakly dominated by B, and F is weakly dominated by D, which rules out any NE in which players mix.*

(c) Now consider the game $G(\infty)$, which consists of the stage game G repeated infinitely many times. Assume that players discount future payoffs with factor δ which is very close to one. Define the average payoff of player $i \in \{1, 2\}$ as $(\sum_{t=1}^{\infty} \delta^{t-1}\pi_{i,t})(1-\delta)$, where $\pi_{i,t}$ refers to player i's payoff in period t.

Describe, either graphically or in words, the set of average payoffs that can be achieved as part of an SPNE of $G(\infty)$ (*NOTE: here you should consider all possible SPNEs, but you do not need to explicitly solve for them*). Does an SPNE exist that gives an average payoff to both players that is at least as high as their payoff from e(1) in part (a)? If so, solve for such an SPNE.

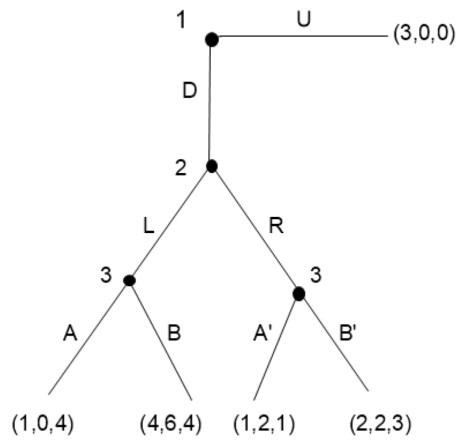
SOLUTION: Graphically, following the slides of Lecture 8, we have:



In words, this is the set of feasible payoffs (i.e. all convex combinations of payoffs in the stage game) in which player 1 earns at least 2 and player 2 earns at least 1 (i.e.

at least their payoffs from a NE of the stage game). Yes, there exists such an SPNE. Consider the strategy profile where player 1 plays B and player 2 plays D in every period, regardless of the history. This strategy profile is a SPNE, because it implies NE play in every subgame. Player 1 earns an average payoff of 3, and Player 2 earns and average payoff of 5. This is at least as high (more specifically, it is equal to) the players' payoffs from e(1) = (B,D), described in part b.

2. Consider the following game:



   (a) Is it a dynamic or a static game? Is it a game of complete or incomplete information?

   SOLUTION: It is a dynamic game of complete information.

   (b) How many pure strategies does each player have to choose from? (*i.e. what is the cardinality of each player's strategy set?*)

   SOLUTION: Player 1 has two strategies to choose from: U and D. Player 2 has two strategies to choose from: L and R. Player 3 has four strategies to choose from: AA', AB', BA', BB'.

   (c) Find all pure strategy Subgame Perfect Nash Equilibria (SPNE).

   SOLUTION: There are two pure strategy SPNE: (D,L,BB') and (U,R,AB'). Notice that Player 3 is indifferent about playing A and B, conditional on reaching his left-most decision node, which is what leads to multiple equilibria.

   (d) Find one pure strategy Nash Equilibrium (NE) that is not subgame perfect.

   SOLUTION: There are many strategy profiles which are NE but are not subgame perfect: (D,L,BA'), (U,L,AA'), (U,L,AB'), (U,R,AA'), (U,R,AB'), (U,R,BA'), (U,R,BB').

3. Two consumers are considering whether to buy a product that exhibits network effects. The payoff from buying depends on the choice of the other consumer. That is, for each consumer $i \in \{1, 2\}$, the payoff U_i from buying depends on three terms: the consumer's

type, θ_i, which represents his intrinsic valuation of the product; a potential network payoff $\lambda > 0$, which consumer i only obtains if consumer $j \neq i$ also buys; and the price p. Specifically, buying yields $U_i = \theta_i + \lambda - p$ if consumer j also buys, or $U_i = \theta_i - p$ if consumer j does not. Not buying the product gives a payoff of zero. Each consumer's type is either $\theta^L = 0$, $\theta^M = 2$, or $\theta^H = 5$, where each type is equally likely. Type is private information. For all parts of this question, you can assume the following parameter values: $\lambda = 3$ and $p = 9/2$.

(a) Suppose consumers must simultaneously decide whether or not to buy, so the strategic situation they face can be seen as a static game of incomplete information. Find the Bayes-Nash equilibrium of this game. (*HINT: do any types have a strictly dominant strategy*?).

SOLUTION: Type θ^H has a strictly dominant strategy to buy, since $\theta^H - p = 5 - 9/2 > 0$. Type θ^L has a strictly dominant strategy not to buy, since $\theta^L + \lambda - p = 0 + 3 - 9/2 < 0$. It remains to check whether type θ^M will buy in equilibrium. In a candidate equilibrium where both θ^M and θ^H buy, but θ^L does not, the expected payoff for θ^M from buying is $\theta^M + (2/3)\lambda - p = 2 + (2/3)3 - 9/2 < 0$. This is less than zero, so type θ^M has an incentive to deviate, and hence this cannot be an equilibrium. It follows that in equilibrium, type θ^H buys, but types θ^M and θ^L do not, for both consumers i and j. Beliefs are simply given by the prior, that each type is equally likely.

(b) Now consider the following modified situation. Consumer i moves first by deciding whether or not to buy the product. Consumer j observes the decision of consumer i, and then decides whether to buy the product herself. As a result, the strategic situation the consumers face can be seen as a dynamic game of incomplete information. Find the Perfect Bayesian equilibrium of this game.

SOLUTION. Consumers i and j will buy no matter what if they are type θ^H, and not buy no matter what if they are type θ^L, since these strategies are strictly dominant for these types. If consumer j is of type θ^M, then he will buy if and only if he observes that consumer i chose to buy, because $\theta^M + \lambda - p = 2 + 3 - 9/2 > 0$ and $\theta^M - p = 2 - 9/2 < 0$. As a result, consumer i realizes that buying will give him an expected network payoff of $(2/3)3$, since consumer j will respond by buying with probability $2/3$. Still, consumer i prefers not to buy himself, if he type θ^M, since $\theta^M + (2/3)\lambda - p = 2 + (2/3)3 - 9/2 < 0$. This means that equilibrium strategies are as follows: consumer i buys if and only if his type is θ^H; consumer j buys if and only if either (i) his type is θ^H, or (ii) his type is θ^M and consumer i bought. Consumer i's beliefs about consumer j are given by the prior, that each type is equally likely. (Note: consumer j's beliefs about consumer i are not relevant in this setting, although he could in principle update his beliefs after observing j's purchase decision.)

(c) One way to interpret part (a) is that the firm follows a 'sprinkler' marketing approach, where it launches the product simultaneously in multiple markets. One way to interpret part (b) is that the firm follows a 'waterfall' marketing approach, where it launches the product sequentially across markets. Given these interpretations, and using your answers in parts (a) and (b), argue whether a 'sprinkler' or a 'waterfall' approach is more profitable in this situation, and briefly give the intuition why this is the case.

SOLUTION: Part (a) implies that each consumer buys with probability 1/3, giving expected revenues of $(1/3 + 1/3)(9/2) = 3$. Part (b) implies that consumer i buys with *probability* $1/3$ *and consumer* j *buys with probability* $(1/3)(2/3) + (2/3)(1/3) = 4/9$, *giving expected revenues of* $(1/3{+}4/9)(9/2) = 7/2$. *Hence, in this particular situation, a waterfall approach is more profitable. Allowing consumer* j *to observe consumer* i *can convince* j *to buy, if he is an intermediate type, and if he sees that the product is popular (i.e. if he sees that* i *chose to buy, which occurs with positive probability). In this sense, following a waterfall approach may help a firm to exploit network effects by allowing 'success to breed success', as an early purchase can then help get the bandwagon rolling.*